A CONSTRUCTION OF NON-CYCLIC NORMAL DIVISION ALGEBRAS*

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1. *Introduction.* We know now that every normal division algebra over an algebraic number field is a cyclic (Dickson) algebra. This result was proved by highly refined arithmetic means† and the proof cannot be extended to obtain a like result for algebras over a general field. The very important question of whether or not any non-cyclic algebras exist has thus remained unanswered up to the present.

I shall give a construction of non-cyclic algebras of order sixteen over a function field‡ in this paper. These algebras will be proved to be normal division algebras; they furnish the first example in the literature of linear associative algebras of division algebras definitely known to be not of the Dickson type.

2. *A Type of Division Algebra.* Let K be a non-modular field and $K(z)$, $z^2=\Delta$ in F, be a quadratic field over K, so that Δ is not the square of any quantity of K. I have proved§ the following proposition.

LEMMA 1. *Let A be a division algebra over K. Then $A\times K(z)$ is a division algebra if and only if A contains no sub-field $K(z_0)$, $z_0^2=\Delta$, equivalent to $K(z)$.*

We shall restrict further attention to fields

$$K=F(u,v),$$

where F is any real number field and u and v are independent indeterminates. Then K is the field of all rational functions with

* Presented to the Society, April 9, 1932.

† A proof by H. Hasse (to whom are due the arithmetic considerations) and by myself will appear very soon in the Transactions of this Society.

‡ Algebras of the type constructed here were first considered by R. Brauer who proved (falsely) that they were all division algebras. See Section 4 of this paper for a discussion which points out the error in Brauer's work and which gives simple examples of Brauer algebras not division algebras. (See also, however, a footnote on p. 455, added in proof.)

§ This theorem is a consequence of a result of L. E. Dickson, *Algebren und ihre Zahlentheorie*, pp. 63–64. For my application to prove the above Lemma see this Bulletin, April, 1931, pp. 301–312; p. 309.