ON THE ZEROS OF CERTAIN POLYNOMIALS RELATED TO JACOBI AND LAGUERRE POLYNOMIALS*

BY W. LAWTON†

1. *Introduction.* We consider the polynomials defined as follows:

$$(1)\qquad J_n(x, \alpha, \beta) \equiv x^{1-\alpha}(1 - x)^{1-\beta}\frac{d^n}{dx^n}\left[x^{n+\alpha-1}(1 - x)^{n+\beta-1}\right],$$

$$(2)\qquad L_n(x, \alpha) \equiv x^{1-\alpha}e^{x}\frac{d^n}{dx^n}\left[e^{-x}x^{n+\alpha-1}\right],$$

where α and β are arbitrary real numbers. If $\alpha, \beta > 0$, they are known respectively as Jacobi and Laguerre polynomials, satisfying the following orthogonality relations:

$$\int_0^1 x^{\alpha-1}(1 - x)^{\beta-1}J_m(x)J_n(x)dx = 0,$$

$$\int_0^\infty e^{-x}x^{\alpha-1}L_n(x)L_m(x)dx = 0,$$

$$(\alpha, \beta > 0; m, n = 0, 1, \cdots ; m \neq n).$$

From these relations it can be shown that all the zeros of the functions $J_x(x, \alpha, \beta)$ and $L_n(x, \alpha)$ are real, distinct, and lie respectively inside $(0, 1)$, $(0, \infty)$.

The following differential equations are also well known:

$$(3)\qquad x(1 - x)J_n''(x, \alpha, \beta) + \{\alpha - (\alpha + \beta)x\}J_n'(x, \alpha, \beta) + n(n - 1 + \alpha + \beta)J_n = 0, \qquad (\alpha, \beta > 0),$$

$$(4)\qquad xL_n''(x, \alpha) + (\alpha - x)L_n'(x, \alpha) + nL_n(x, \alpha) = 0.$$

Since (3) and (4) represent identical relations between the coefficients of $J_n(x, \alpha, \beta)$ and $L_n(x, \alpha)$ respectively which are polynomials in α, β or in α respectively, we conclude that the differential equations still hold, if $\alpha, \beta \leqq 0$.

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† Harrison Fellow in Mathematics, University of Pennsylvania.