AN EXAMINATION OF SOME CUT SETS OF SPACE*

BY C. H. HARRY

PART I

The purpose of this paper is to examine some pairs of points which are cut sets of a locally connected, locally compact, separable, and connected metric space S which has no single cut point. Under such an hypothesis the following statement will be proved.

If L is the set of all points (x) such that x together with some point y_x separates two fixed points a and b of the space S, then $L+a+b$ is closed and compact.†

By the pair (x, y) separating a and b is meant that there exists at least one separation $S_a+S_b=S-x-y$ such that no point of S_a is a point or limit point of S_b and no point of S_b is a limit point of S_a, where $a \subset S_a$ and $b \subset S_b$.

Two properties of S used in the proof are the following:

I. Between a and b there exists at least one pair of arcs T_x and T_y having just their end points a and b in common.‡

II. If X is any closed set, every component of $S-X$ is an arcwise connected open set with at least one limit point in X.§

Properties of simple arcs which are used are the following:

III. If x is any point of an arc ab, then ab may be written as the sum of two arcs ax and xb having just x in common.

IV. The points of an arc ab may be ordered. If it is assumed that a precedes b, $a \propto b$, the ordering gives the following relations:

---

* Presented to the Society, September 9, 1931.

† This result is analogous to the theorem of G. T. Whyburn, this Bulletin, vol. 33 (1927), p. 685, to the effect that if, in any locally connected and metric continuum S, K is the set of all points separating two fixed points a and b, then $K+a+b$ is closed and compact. See also R. L. Wilder, this Bulletin, vol. 34 (1928), p. 649.

‡ See G. T. Whyburn, Proceedings of the National Academy of Sciences, vol. 13 (1927), pp. 31–38; and W. L. Ayres, American Journal of Mathematics, vol. 51 (1929), pp. 577–594. For a short proof of this theorem see G. T. Whyburn, this Bulletin, vol. 37 (1931), p. 429.

§ R. L. Moore, Mathematische Zeitschrift, vol. 15 (1922).