5. *Bearing on the Nature of the Theory of Deduction.* The fact that proposition (viii) cannot be derived from the theory of deduction has an important bearing on the nature of that theory. The theory of deduction has been designed as "the calculus of propositions." Proposition (viii) is a well known proposition in the classic logic of propositions; the theory cannot yield this proposition; and so the theory cannot serve as "the calculus of propositions."

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A SUFFICIENT CONDITION FOR THE EXISTENCE OF A DOUBLE LIMIT

BY J. A. CLARKSON

In the elementary theory of limits it is often emphasized that the existence of a unique limit for a single-valued function $f(x, y)$ as the point $P(x, y)$ approaches $Q(a, b)$ along every straight line through Q does not imply the existence of the double limit

$$\lim_{\substack{x \to a \\ y \to b}} f(x, y). \tag{1}$$

As early as 1873 Thomae* gave an example to illustrate this fact.

The question then naturally arises: Is the existence of a unique limit as P approaches Q along some more extensive class of curves sufficient to insure the existence of (1)? This question is immediately answered by the following theorem.

THEOREM. *If $f(x, y)$ has a unique limit L as $P(x, y)$ approaches $Q(a, b)$ along every curve having a tangent at Q, the double limit* (1) *exists.*

PROOF. Suppose, if possible, that it does not. Then there exists an $\epsilon > 0$ such that in any circle about Q there are points p for which

$$|f(p) - L| > \epsilon. \tag{2}$$

We denote by E the set of all such points.

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* J. Thomae, *Abriss einer Theorie der complexen Functionen*, 2d ed., Halle, 1873, p. 15.