ADJOINT SYSTEMS IN THE PROBLEM OF MAYER UNDER GENERAL END-CONDITIONS*

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1. *Introduction.* Necessary conditions in the problems of Lagrange and Mayer with variable end points, analogous to the Euler equations and transversality condition in the ordinary problem of the calculus of variations in the plane, have recently been given by Morse and the author.† The methods used in the derivation of these conditions, as well as those used by Bliss‡ in his derivation of somewhat more complicated transversality conditions in the same problem, consist primarily of certain proofs which more properly belong to the theory of differential equations.

In the present paper, certain adjoint relationships in the problem hitherto unnoticed are pointed out which make it possible to derive the necessary conditions mentioned above merely by a consideration of the compatibility of the adjoint system of a certain set of differential equations and boundary conditions. In fact, in the anormal case it is shown that these necessary conditions form an adjoint system of the differential and terminal equations of variation. The normal case is treated essentially by making it the abnormal case of a slightly different problem. Such considerations have the advantage of referring a major part of the proofs to the theory of differential equations, and thus greatly simplifying them.

2. *The Definition of the Adjoint System.* Consider the set of m linear homogeneous differential equations of the first order§ in the n variables $\eta_1, \cdots, \eta_n$,

$$L_\beta(\eta) \equiv p_{\beta i}\eta_i' + q_{\beta i}\eta_i = 0, \quad m \leqq n, \tag{1}$$

$$(\beta = 1, \cdots, m;\ i = 1, \cdots, n),$$

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† Morse and Myers, Proceedings of the American Academy of Arts and Sciences, vol. 66 (1931), pp. 235–253.

‡ Bliss, Transactions of this Society, vol. 19 (1918), pp. 305–314.

§ The usual convention of summation with respect to an index repeated in the same term is used throughout.