A Course of Analysis. By E. G. Phillips. Cambridge University Press, 1930. vii+361 pp.

This book is based on the first part of a course of lectures on analysis given by the author to mathematical honors students in the University College of North Wales. The presentation follows the traditional English manner, clear, rich in content, and compact. The topics considered cover a wide range and their choice and arrangement are well calculated to arouse and hold the interest of the student. The work is intended for students of moderate mathematical maturity who have been prepared by a standard course in calculus.

The first chapter is devoted to an outline of the logical foundations of analysis and the structure of the number system. Chapters on sequences and limits and the theory of continuous functions are followed by an exposition of the elements of the differential calculus. Here the distinction between derivable and differentiable is introduced. This finds an important application later in the chapter on functions of several variables. A brief introductory chapter on series designed to provide a basis for later developments is followed by an interesting chapter on the inequalities of Hölder, Minkowski, Jensen and others. Inequalities are treated throughout the work thoroughly and elegantly, a feature of special value.

The theory of integration is limited to the Riemann integral. This limitation can hardly be avoided in an introductory volume. The distinction between a primitive and an indefinite Riemann integral is carefully illustrated. Infinite integrals are briefly treated.

After extending results previously obtained to functions of several variables the author presents in the tenth chapter a readable discussion of implicit functions with an application to the theory of the exponential function and an adequate treatment of transformations and jacobians. Chapters on multiple integration with particular reference to classical formulas of mathematical physics follow. The work concludes with a short chapter on power series and functions defined by power series, with an application to the theory of the circular functions.

Lists of exercises at the chapter ends contribute materially to the value of the work as a text. As many of the exercises are beyond the power of average students, it would be beneficial to have additional exercises for practice.

In a work of this scope and brevity it is difficult to secure perfect balance. Some subjects such as infinite integrals are too briefly treated, others, for example the integration of non-uniformly convergent sequences, receive more attention than is desirable in an introductory course. While misprints are rare, the author has frequently failed in his effort to present the student with an accurate exposition. Gehman (American Mathematical Monthly, vol. 38, pp. 166–167) has pointed out a number of slips and commented on the frequent errors in the statement of theorems, particularly mean-value theorems. He calls attention to a fallacious proof on page 32 of the theorem of Weierstrass. Another notable error occurs in the proof of Theorem 3, page 344; it invalidates the demonstration of Abel's theorem on power series. In theorems involving transformations of multiple integrals, Phillips often fails to assume that the jacobian does not change sign throughout the region of integration.