HEXAGONAL SYSTEMS OF SEVEN LINES IN A PLANE*

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1. *Introduction.* This paper concerns the determination of the non-equivalent systems of seven real lines in a plane when no three of the lines are co-punctual, and the investigation is limited to that subdivision of the problem where, in a system of seven lines, some six form a convex hexagon. It is shown that exactly eight non-equivalent arrangements exist, and incidentally the integers showing the numbers of polygons of 3, 4, 5, 6, 7 sides which occur in each of the eight systems are tabulated. The method, developed by Professor H. S. White, of the unique characterization of a line by means of the contiguous line-segments in a system, has been used to determine a "mark" for every line in the eight systems. The seven marks of a system are employed to prove the non-equivalence of systems and to determine the substitution connecting two equivalent systems.

2. *Basic Hexagon.* Six lines in a plane, no three in any point, form 30 segments, and for the hexagonal subdivision here considered, divide the plane into one hexagon, six triangles, and nine quadrilaterals. For easier visualization, an irregular hexagon with sides produced indefinitely and with all 15 intersections in the finite plane is considered. The 30 segments of the six lines may be assigned to three classes, namely, 6 primary, separating the hexagon from the triangles, 12 secondary, separating the triangles from quadrilaterals, 12 tertiary, separating, each, two quadrilaterals. The nine quadrilaterals are separable into two types: (1) six quadrilaterals with sides two adjacent secondary segments and two adjacent tertiary segments, (2) three quadrilaterals with all sides tertiary segments. The 30 segments separate into five continuous broken lines as follows: the 6 primary segments bound the hexagon, the 12 secondary segments surround the six triangles, and the 12 tertiary segments separate into the three boundaries of the three quadrilaterals of the second type.

3. *Notation and Method.* When, in a set of seven lines, any six form a hexagon, that hexagon is utilized as unique initial figure.

* Presented to the Society, October 31, 1931.