A TRIAD OF RULED SURFACES DEFINED BY RECIPROCAL POLARS*

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As an application of the theory of triads of ruled surfaces† in projective differential geometry, we shall consider here a system defined as follows. Let l_{yz} be any line element of a general ruled surface R_{yz}, the points P_y, P_z being the flecnodes of l_{yz}. The defining system of differential equations for R_{yz} will have the form

$$(1)\qquad \begin{aligned} y'' + p_{12}z' + q_{11}y + q_{12}z &= 0,\\ z'' + p_{21}y' + q_{21}y + q_{22}z &= 0, \end{aligned}$$

where differentiation is with respect to a parameter x and where $p_{12}' = 2q_{12}$, $p_{21}' = 2q_{21}$.

The planes osculating the flecnode curve C of R_{yz} at P_y, P_z intersect in a line $l_{\psi\phi}$, the points P_ψ, P_ϕ of which are those in which $l_{\psi\phi}$ is cut by the tangents to C at the respective points P_z, P_y. The expressions for ψ, ϕ are

$$(2)\qquad \psi = p_{12}z' + q_{12}z, \quad \phi = p_{21}y' + q_{21}y.$$

The polar reciprocal of $l_{\psi\phi}$ with respect to the linear complex which osculates R_{yz} along l_{yz} is the line $l_{\eta\theta}$, the points P_η, P_θ of which are given by the expressions

$$(3)\quad \eta = p_{12}z' + p_{12}p_{21}y + q_{12}z, \quad \theta = p_{21}y' + q_{21}y + p_{12}p_{21}z.$$

The points P_y, P_ψ, P_η are collinear, as are also the points P_z, P_ϕ, P_θ.

A set of three lines l_{yz}, $l_{\psi\phi}$, $l_{\eta\theta}$, as thus defined, corresponds to each value of the parameter x and determines in this way three ruled surfaces R_{yz}, $R_{\psi\phi}$, $R_{\eta\theta}$. From (1), (2) and (3) we obtain the defining system of differential equations for this triad of ruled surfaces.‡ It is

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† A. F. Carpenter, *Triads of ruled surfaces*, Transactions of this Society, vol. 29 (1927), pp. 254–275. Hereafter denoted by the symbol T.

‡ T, p. 256.