ON SURFACES IN SPACE OF r DIMENSIONS

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Consider a surface F^n of order n in r-space. Let it be the complete intersection of $q \leqq r-2$ varieties $V_{k_1}^{n_1}, V_{k_2}^{n_2}, \cdots, V_{k_q}^{n_q}$ of orders $n_1, n_2, \cdots, n_q$ and of dimensions $k_1, k_2, \cdots, k_q$, respectively, where

$$\text{(A)} \qquad \begin{aligned} 3 \leqq k_1, k_2, \cdots, k_q \leqq r-2, \\ k_1 + k_2 + \cdots + k_q = r(q-1) + 2. \end{aligned}$$

Project F^n on an S_3. The projection F'^n has a number of characteristics of which we note the following six: n, its order; a, the order of its tangent cone; b, the order of its double curve; j, the number of its pinch-points; t, the number of its triple points; and m, its class. If we project F^n on an S_4, the projection has a finite number, d, of improper double points. We shall call these seven characteristics, of which n, a, t, m are often regarded as essential, the characteristics of F^n, and they are known to satisfy the following relations:*

$$\text{(B)} \qquad \begin{aligned} a + 2b = n(n-1), \quad j + 2d = n(n-1) - a, \\ j = \tfrac{1}{4}[a(3n-4) - n(n-1)(n-2) + 6t - 2m], \\ d = \tfrac{1}{8}[n(n-1)(n+2) - 3an - 6t + 2m]. \end{aligned}$$

For $r=5$, $q=3$, $k_1=k_2=k_3=4$, F^n is the intersection of three hypersurfaces in S_5. Formulas for its characteristics are known† and they are symmetric functions of the orders of the hypersurfaces. In this note we present analogous formulas for the same characteristics of F^n for r general and for $q \leqq r-2$. As the method of obtaining these formulas is familiar and has been applied by the writer time and again to similar enumerative problems,‡ we shall here omit all demonstration.

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* Severi, *Intorno ai punti doppi impropri di una superficie generale dello spazio a quattro dimensioni, e a'suoi punti tripli apparenti*, Rendiconti di Palermo, vol. 15 (1901), pp. 33–51.

† B. C. Wong, *On surfaces in spaces of four and five dimensions*, this Bulletin, vol. 36 (1930), pp. 681–686. Opportunity is here taken to correct an error in the formula for T'' on page 685 of this paper. The formula should read

$$T'' = \tfrac{1}{2}\lambda\mu\nu(\lambda-1)(\mu-1)(\nu-1)(\mu\nu+\nu\lambda+\lambda\mu-2\lambda-2\mu-2\nu).$$

‡ B. C. Wong, loc. cit., and also the paper *On the number of apparent double points of r-space curves*, this Bulletin, vol. 37 (1931), pp. 421–423.