THE PLANE FIGURE OF SEVEN REAL LINES*

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1. *Introduction.* A set of real lines, finite in number, in the plane of projective geometry, divides the plane into convex polygons, each bounded by segments of those lines. If no three lines meet in a point, then three lines form 4 polygons (triangles); four lines, 7 polygons; n lines, $(n^2-n+2)/2$ polygons. For any given diagram of this sort, the triangles, quadrilaterals, etc., may be counted; then if the lines are allowed to move freely in the plane, every polygon will retain the same number of sides until three or more points of intersection (or two pairs) come to coincide. Exclude this situation, and we have as invariants the integers showing the numbers of polygons of 3, 4, 5, $\cdots$, n sides; also a scheme showing contiguities. We shall exhibit such schemes for a set of 7 lines; and inquire how many kinds of (non-equivalent) sets exist, when a one-to-one relation between lines and polygons of two sets constitutes equivalence.

2. *Unique Sets*, $n=3, 4, 5$. Since central projection is a particular kind in the group of transformations that we here admit, and since four lines in a plane are projective to any other set of four—barring cases where three lines are copunctual—the arrangement of any such set is typical of all. Three lines form four triangles; three of them have infinitely long boundaries, but that is projectively of no account; and each is adjacent to all the others. A fourth line intersects three segments exterior to one of these triangles, say T_1, and divides each of the other triangles into a quadrilateral adjacent to T_1 and a triangle having no sides, but only one vertex, in common with T_1. Otherwise stated (Fig. 1), *four lines in a plane, no three in any point, constitute twelve segments bounding four triangles and three convex quadrilaterals. Each quadrilateral is adjacent to all four triangles, but two triangles have in common only one vertex, while two quadrilaterals have in common two opposite vertices.*

Five lines are not necessarily *projective* to an arbitrarily se-

* Presented to the Society, October 31, 1931.