IDEALS IN LINEAR ALGEBRAS*

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1. *Introduction.* In an algebraic field, and indeed in every commutative domain of integrity, the problem of the greatest common divisor and the problem of unique factorization into primes are equivalent. That is, the property that every pair of numbers in the domain of integrity have a greatest common divisor unique up to a unit factor implies the property that factorization be unique save for unit factors and order of multiplication. In those commutative domains of integrity in which these properties do not hold, the introduction of Dedekind ideals gives an extended system in which they do hold.

In a domain of integrity of a non-commutative algebra, the existence of the greatest common right divisor and of the greatest common left divisor does not insure unique factorization. The two problems have diverged and it seems very improbable to the writer that the same type of ideal system will handle both problems.

The Dedekind ideal is by its very definition connected with the problem of the greatest common divisor. This theory, including the theory of the class number, seems to extend practically intact to the non-commutative case. From this standpoint the theory of ideals in linear algebras is successful.

Several writers have attempted to apply Dedekind ideals to the problems of unique factorization, and in every case the result has been of dubious value. The difficulty has always been in finding an adequate definition for ideal multiplication. In every case so many important properties of ideals have failed to generalize that the theory is practically without substance. It would seem that the enlargement of the domain to permit unique factorization will have to be accomplished through other agencies than the Dedekind ideal.

2. *Domains of Integrity.* The elements of a rational algebra $\mathfrak{A}$ of order n are given by

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