WEATHERBURN'S SECOND VOLUME

Differential Geometry of Three Dimensions. Volume II. By C. E. Weatherburn. Cambridge University Press, 1930. xii+239 pp.

The first volume of this work (see this Bulletin, vol. 34 (1928), pp. 785–786) covered the fundamentals of metric differential geometry of curves and surfaces. The present volume, though containing certain classical material supplementing that of the first volume, is primarily devoted to a consequential exposition of the author's published contributions to the subject.

The treatment in both volumes is in terms of vectors. But, whereas the first volume employs, except in the last chapter, on differential invariants, only the algebra of vectors, the second volume uses also, and to a great extent, the differential and integral calculus of vectors. Moreover, dyadics are introduced in the middle of the volume and are used to good effect throughout the later chapters.

Chapters 2, 3, and 8 contain the author's work on families of curves on a surface. In Chapters 5 and 6 the calculus of vectors referred to curvilinear coordinates in space is developed and applied to families of surfaces. Chapters 10, 11, and 12 have to do with transformations, small deformations, and applicability of surfaces. In Chapter 13 are found the author's contributions to the theory of curvilinear congruences, and in Chapter 9 those bearing on the parallelism of Levi-Civita.

In treating the family of curves $\phi(u, v) = \text{const.}$ on a surface, the author makes extensive use of the function $\psi = 1/(\Delta_1\phi)^{1/2}$. Since the distance between the curve $\phi = \phi_0$ and the curve $\phi = \phi_0 + \Delta\phi$, measured along the orthogonal trajectories of the curves $\phi = \text{const.}$, is approximately $\psi\Delta\phi$, he calls ψ the distance function for the given family of curves and the curves $\psi = \text{const.}$ the lines of equidistance for this family. Similarly, in treating a family of surfaces in space $\phi(u, v, w) = \text{const.}$, he introduces the distance function $\psi = 1/(\Delta_1\phi)^{1/2}$, the surfaces of equidistance $\psi = \text{const.}$, and the lines of equidistance $\psi = \text{const.}$, $\phi = \text{const.}$

The distance function enables the author to simplify greatly the formal work. His skilful use of it in connection with the study of ruled surfaces in Chapter 4 is particularly to be commended. There is, however, an important point in connection with it which he fails to mention. Though the function itself is invariantly connected with the given function ϕ, the lines of equidistance for a given family of curves on a surface and the surfaces of equidistance for a given family of surfaces are not fixed respectively by the family of curves and the family of surfaces; they vary with the choice of the function ϕ employed to define the given family. On the other hand, the lines of equidistance for a family of surfaces are uniquely determined by the family itself.

In applying dyadics to a family of curves on a surface, the author introduces what he calls the tendency, the moment, and the swerve of the family in a given direction. If $\boldsymbol{t}$ is the unit vector tangent to the curve of the family passing through a point P and $\boldsymbol{a}$ is the unit vector in the given direction at P, these