MAPS OF CERTAIN CYCLIC INVOLUTIONS ON TWO-DIMENSIONAL CARRIERS

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1. *Introduction.* The following paper derives the fundamental properties of the involutions on an algebraic surface which have but a finite number of invariant points. Except for a few particular cases, they cannot be regarded as subcases of those having a curve of invariant points; they require one more equation for their definition, analogous to the singular correspondences on algebraic curves. They exist only on surfaces having particular moduli.

2. *Discussion of* I_n. Consider two surfaces $F(x)=0$ and $\Phi(x')=0$ with the property that any point P on $F(x)=0$ uniquely fixes a point P' on $\Phi(x')=0$ and, conversely, the point P' fixes n points $P_1 \equiv P, P_2, \cdots, P_n$ on F. There is thus set up an $(n, 1)$ correspondence between the points of $F=0$ and $\Phi=0$. Now any one of the n points $P_1, \cdots, P_n$ on $F=0$ definitely determines the whole group of n points to which it belongs. Hence, it will be said that F *contains an involution* I_n of order n, and that this I_n *belongs to* $\Phi(x')=0$.

There are two kinds of involutions; F may contain one or more curves, each point of which contains two or more coincidences of these n points $P_1, \cdots, P_n, P_i=P_k$. Such curves are called *curves of coincidences*. The surface $\Phi(x')=0$ then contains a locus of branch points in (1, 1) correspondence with the curve of coincidences on F. The other kind of involution is such that F has only a finite number of coincident points. Thus, $\Phi(x')$ has in this case exactly the same number of branch points.

If $\Phi(x')=0$ is a rational surface, or a plane, I_n is said to be *rational*. If $F(x)=0$ is rational, $\Phi(x')=0$ must be rational.* The converse is not true.

In this paper only I_n on $F(x)=0$ with a finite number of coincident points will be considered. Such an I_n can be gener-

* Castelnuovo, Mathematische Annalen, vol. 44 (1894), pp. 125–155.