A NOTE ON CYCLIC ALGEBRAS OF ORDER SIXTEEN

BY A. A. ALBERT*

1. *Introduction.* In a recent paper† I considered cyclic (Dickson) algebras of order sixteen generated by a cyclic quartic field Z and a quantity γ in the reference field F. It was proved there that if the algebra A were a division algebra and if γ^2 were the norm of a quantity of Z, so that the Wedderburn *norm condition* would not be satisfied, then A would be the direct product of two generalized quaternion algebras. It was not proved, however, that such division algebras existed.

R. Brauer has recently‡ proved that there exist normal division algebras which are direct products of two generalized quaternion algebras. In the present note I give an example of a *cyclic* algebra over the Brauer reference field for which the *norm condition* is not satisfied, therefore completing the theory of the previous paper.

2. *The Example.* Let $F=R(\xi, \eta)$, where ξ and η are indeterminates and R is the field of all rational numbers. This is the reference field of the algebras of Brauer. We shall use the notations of my paper (loc. cit.), Theorem 3. It was proved there that a necessary and sufficient condition that a direct product of two generalized quaternion algebras over F be a division algebra is that the connected form

$$\tau x_1^2 + \sigma x_2^2 - \sigma\tau x_3^2 - (\gamma x_4^2 + \rho x_5^2 - \rho\gamma x_6^2), \tag{1}$$

in the variables $x_1, \cdots, x_6$ in F be not a null form. We shall take

$$\sigma = -2\xi^3, \quad \rho = \eta, \quad \gamma = -1, \quad \tau = \alpha, \tag{2}$$

where α is a rational number not the square of a rational number.

Suppose that $\alpha = \nu^2$, where ν is in F. Then we may write

$$\nu = bc^{-1},$$

---

* Presented to the Society, September 9, 1931.

† This Bulletin, vol. 37 (1931), pp. 301–312.

‡ Mathematische Zeitschrift, vol. 31 (1930), §5. Brauer's example is that of an algebra not necessarily a cyclic algebra and it would probably be difficult to prove it cyclic even if this were the case. Also his proof that the algebra is a division algebra is essentially different from ours.