A NON-DENSE PLANE CONTINUUM†

BY J. H. ROBERTS

The author has shown ‡ that if M denotes a square plus its interior in a plane S, then there exists an upper semi-continuous collection § G of mutually exclusive non-degenerate subcontinua of M filling up M and such that G is homeomorphic with M. The present paper gives a continuum M *which contains no domain* yet which has the above property.

Let I denote the interior of a square J in a plane S. Let G be an upper semi-continuous collection of mutually exclusive non-degenerate continua filling $J+I$ such that G is homeomorphic with $J+I$. Since no element of G separates S it follows‖ that if S' denotes the collection consisting of the elements of G and the *points* of S not belonging to any element of G, then S' corresponds to S under a continuous one to one correspondence U, and G corresponds to $J+I$. Let G^* denote the subcollection containing every element of G which contains a point of J. The set G^* is a simple closed curve. Moreover every element of G^* has in common with J either an arc or a single point. Then there exists¶ a continuous one to one correspondence U_1 between G^*

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‡ *On a problem of C. Kuratowski concerning upper semi-continuous collections*, Fundamenta Mathematicae, vol. 14 (1929), pp. 96–102.

§ For a definition of this term, and of the notion *limit element*, see R. L. Moore, *Concerning upper semi-continuous collections of continua*, Transactions of this Society, vol. 27 (1925), pp. 416–428.

‖ See R. L. Moore, loc. cit., Theorem 22.

¶ This may be shown as follows. Let $s_1, s_2, s_3, \cdots$ denote the maximal arcs which are subsets of J and which belong to some element of G^*. For each i and j $(i \neq j)$ the set $s_i \cdot s_j$ is vacuous. Let $v(s_i)$ denote the length of the interval s_i, and $v(J)$ the length of J. Suppose first that $v(J)-\sum_{i=1}^{\infty}v(s_i)$ is a positive number e and let d be a positive number less than e. A sequence of segments $t_1, t_2, t_3, \cdots$ can be defined inductively so that (1) for each i there is a j such that t_j contains s_i, (2) no two of the segments $t_1, t_2, t_3, \cdots$ have any point in common, and (3) $\sum_{i=1}^{\infty}v(t_i)$ is less than $v(J)-d$. Since the point set $J-\sum_{i=1}^{\infty}t_i$ has positive measure it is uncountable. Now the curve J can be transformed into itself in such a way that the sum of the lengths of the images of the intervals $s_1, s_2, s_3, \cdots$ is less than the length of J. Hence in any case there is a set of segments