ANALYTIC FUNCTIONS AND MATHEMATICAL PHYSICS*

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1. *Some Properties of Analytic Functions.* We begin by reviewing briefly some fundamental points in the theory of analytic functions in a form which will be convenient for further references. Departing slightly from customary notations, we shall write $w=v+iu$, and we shall consider the theory primarily as the theory of a system of differential equations

$$(1) \qquad \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0, \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

which are called the Cauchy-Riemann equations.

We shall not enter into various fine points which arise in the discussion, but we may mention that the functions u and v, as well as the other functions which appear later, must be assumed to be differentiable, that is, to possess complete differentials in the sense of Stolz.

(a) One point of view often taken in applications is that we have a vector or, rather a vector field, of components (u, v), and that the differential equations express the fact that the rotation (curl) and the divergence of this vector are zero.

Another point of view, which we shall find extremely useful, is that we have *two* vectors f and r, whose components are $f_1=u$, $f_2=v$, and $r_1=v$, $r_2=-u$, respectively, and that the differential equations express the fact that the divergences of both are zero. These two vectors, as the relations

$$(2) \qquad f_1 = -r_2, \qquad f_2 = r_1,$$

show, are perpendicular and of equal length, so that we may say that the theory of analytic functions is the theory of two equal and perpendicular vectors in the plane, with zero divergences.

(b) The differential equations may be considered as integrability conditions. They are equivalent to the vanishing of certain

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