Spektraltheorie der unendlichen Matrizen. Einführung in den analytischen Apparat der Quantenmechanik. By Aurel Wintner. Leipzig, S. Hirzel, 1929. xii+280 pp.

In the Introduction the author defines the main purpose of the book (not quite in accord with the subtitle) to serve as an introduction to the general theory of linear analysis of infinitely many variables. It is stated that, according to the desire of the publishers, the book is designed primarily for beginners. This desire led the author to some compromises as to the choice of material and the character of exposition, of which the most regrettable, although perhaps not entirely unavoidable, is the exclusion of the notion of Lebesgue's integral.

Chapter I is devoted to a rapid survey of fundamental facts of the theory of matrices and bilinear forms in a finite number of variables. We find here a condensed treatment of reduction of matrices to various canonical forms with applications to the theory of hermitian, unitary and normal matrices, of Jacobi's transformation of matrices and its application to hermitian matrices with a simple spectrum. As the author expresses himself, the material is presented here, not always in the simplest and most natural way, but rather in such a way as to permit extension almost without modifications to the theory of infinite matrices. While much might be said in favor of such a method of presentation, it can be hardly considered as the best one for the beginners. Only an experienced and well informed ("kundiger") reader can appreciate many a subtle detail which would puzzle a beginner.

In Chapter II the author gives a discussion of indispensable analytic tools: Stieltjes integrals, properties of sequences of functions of bounded variation (theorems of Helly and Helly-Bray), inversion formulas of Stieltjes and Hilbert, theorems of Grommer and Hamburger. At the end of the chapter the author attempts to give a "gemeinverständlich" report on Hellinger's integrals. No adequate idea of this theory can be given without using the notion of measure, which is being carefully avoided by the author. Hence, in the reviewer's opinion, the book would only gain if the corresponding pages, 106–120, had been omitted.

Chapter III deals with general properties of bounded matrices and of their resolvents: "Faltung" theorems of Hilbert, criteria and "formal" theorems of Toeplitz, theorems of Hellinger and Toeplitz, C. Neumann's series for the resolvent, characterization of the resolvent as an analytic function of the parameter. The treatment is elegant and presents several novel points of interest.

Chapter IV gives a rather condensed and somewhat incomplete discussion of the spectral matrices. A "beginner" will not readily understand the "Hauptsatz über Einzelmatrizen" on page 159 and the subsequent discussion, even if the formula (258) on page 160 did not contain a disturbing misprint (compare with (76) on page 53).

Chapter V is devoted to the existence proof of spectral matrices for various classes of bounded matrices. This chapter contains material of considerable interest and importance. Several results of this chapter are due to the author, among them the elegant treatment of the unitary matrices and their spectral matrices on the basis of the trigonometric moment problem. The reviewer was not able, however, to follow the proof on page 174.