A RATIONAL QUINTIC SURFACE HAVING NO DOUBLE CURVE

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Montesano in his extensive researches on rational quintic surfaces has noted briefly one such surface obtained by applying a quadratic transformation to the quartic surface known as Nöther's third type.* This quintic surface can be obtained by transformation of a rational sextic surface, due also to Montesano. The plane representation of the quintic which results is the same as that given by Montesano; but, aside from the new method of derivation, there are certain features of the surface and the plane representation not mentioned by him, which it is the purpose of this note to consider.

We start with the rational sextic surface that has a quadruple point Q at which are concurrent 3 coplanar double lines and a triple line, which of course does not lie in the plane of the double lines. If we apply a quadratic transformation of the first type whose fixed conic is the triple line and one of the double lines, and whose fixed point is on another of the double lines, we obtain a rational quintic, noted by Montesano,† which has two consecutive skew double lines, and which I have described in a previous paper.‡ If, however, we apply to the sextic a quadratic transformation whose fixed conic consists of two of the double lines and whose fixed point is on the triple line we obtain another rational quintic which is the subject of this paper.

The plane system of the rational sextic surface is the web of curves of order 9 having in common 8 triple points $A_1, \cdots, A_8$ and 3 simple points B_1, B_2, B_3. This set of curves may be designated as the web of nonics $8A^3B_1B_2B_3$. The image of the triple line k is the plane sextic $8A^2B_1B_2B_3$. The image of a double line d_i is the cubic $8AB_i$. To B_i corresponds the residual line on the sextic surface in the plane of k and d_i. The tangent cone to the sextic at the quadruple point Q consists of the 4 planes kd_1, kd_2, kd_3, and $d_1d_2d_3$.

* Rendiconti di Napoli, vol. 40 (1901), p. 100.

† Ibid., vol. 46 (1907), p. 66.

‡ This Bulletin, vol. 34 (1928), p. 761.