ON COMPLEX METHODS OF SUMMABILITY*

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1. *Introduction.* When a method of summability‡ evaluates a complex sequence $\{s_n\}$ to L, it is of interest to know if that method evaluates $\{\mathcal{R}(s_n)\}$ to $\mathcal{R}(L)$, if it evaluates $\{\mathcal{J}(s_n)\}$ to $\mathcal{J}(L)$, and if it evaluates $\{\bar{s}_n\}$ to $\bar{L}$.§ For linear methods of summability,‖ these three questions are easily shown to be equivalent.

To simplify our discussion, we introduce the two following definitions. *A method of summability has property* A *if, corresponding to each sequence* $\{s_n\}$ *which it evaluates, the sequence* $\{\mathcal{R}(s_n)\}$ *is evaluated to the real part of the value of* $\{s_n\}$. *A method of summability has property* B *if, corresponding to each bounded sequence* $\{s_n\}$ *which it evaluates, the sequence* $\{\mathcal{R}(s_n)\}$ *is evaluated to the real part of the value of* $\{s_n\}$.

2. *Failure of Property* B. That a linear regular method may fail to have property B, and hence a fortiori fail to have property A, follows easily from a consideration of the transformation¶

$$(1) \qquad \sigma_n = \tfrac{1}{2}[1 - (-1)^n i]\, s_{n-1} + \tfrac{1}{2}[1 + (-1)^n i]\, s_n$$

which assigns to a given sequence $\{s_n\}$ the value $\lim \sigma_n$ when this limit exists. The bounded sequence $\{x_n\}$ defined by $x_n = 1 + (-1)^n i$ is evaluated to 0 by (1); but $\{\mathcal{R}(x_n)\}$ is evaluated to 1, $\{\mathcal{J}(x_n)\}$ is evaluated to -1, and $\{\bar{x}_n\}$ is evaluated to 2. This

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‡ By a *method of summability*, we mean simply a rule which assigns to each given sequence (or series) of complex numbers either no value or a single value. For example, if we agree to assign to the complex sequence $\{s_n\}$, where $s_n = u_n + iv_n$, u_n and v_n real, the value $3+4i$ if $v_n \neq 0$ for some n and the value 3 if $v_n = 0$ for all n, we have a method of summability.

§ If $w = u + iv$, where u and v are real, we use $\mathcal{R}(w)$, $\mathcal{J}(w)$, and $\bar{w}$ to denote respectively u, iv, and the conjugate $u - iv$ of w.

‖ For definitions of *linearity*, *regularity*, etc., and for necessary and sufficient conditions for regularity, see an expository paper by W. A. Hurwitz, this Bulletin, vol. 28 (1922), pp. 17–36, and the references there given.

¶ Corresponding to a given sequence $s_1, s_2, s_3, \cdots$, we define $s_0 = 0$.