SOME RECENT DEVELOPMENTS IN ABSTRACT ALGEBRA*

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1. *Introduction.* If one should try to define algebra, it might be said that algebra deals with the formal combination of symbols according to prescribed rules. Such formal combinations are, however, obviously fundamental in most branches of mathematics even outside algebra in the ordinary sense. The recognition of this formal element in the mathematical theories has naturally led to an *algebraization*, which can easily be observed in the present state of many domains of mathematics; if one adopts the views of Hilbert, the whole system of mathematics can be formalized in this way.

When a certain number of formal operations have been laid down, a principal problem is to determine the structure of systems which are *closed* with respect to these operations, that is, have the property that any operation on elements again gives an element of the system. In this problem the notion of *isomorphism* is fundamental; two systems S and S' both closed with respect to a given system of operations, are said to be isomorphic or abstractly identical with respect to these operations if there exists a one-to-one correspondence between the elements of S and S', such that any formal combination of elements in S corresponds to the analogous construction with corresponding elements in S'. An isomorphism between elements of the same system is called an *automorphism*. Two isomorphic systems are in algebra considered as equivalent, and it could therefore also be said, that algebra deals with those properties of systems, which are invariant for isomorphisms. Abstract systems and the notion of isomorphisms originated in the theory of finite groups, where the properties of groups were studied independently of the particular representation of the group.

In an algebraic system *equality* is usually defined and satisfies the following axioms:

* An address presented to the Society at the request of the program committee, December 31, 1930.