WHITEHEAD AND RUSSELL'S THEORY OF DEDUCTION AS A MATHEMATICAL SCIENCE*

BY B. A. BERNSTEIN

1. *Introduction.* Whitehead and Russell develop in their *Principia Mathematica* a theory of deduction for "elementary" propositions.† On this theory almost the entire *Principia* rests. From the primitives (the undefined ideas and the postulates) of this theory and from a few other primitives the authors proceed to derive all logic and all mathematics. It is then of great importance to the mathematician and to the logician to understand clearly the mathematical make-up of the theory of deduction. But such understanding is not easy. The novel views of the *Principia*, its strange symbolism, its elaborate structure, its long informal discussions, all combine to hide the real mathematical nature of the theory from the average man interested in mathematics and in logic. The purpose of this paper is to examine the theory critically and to exhibit clearly its true nature as a mathematical science.

My discussion will be based on the formal, the "official" development of the theory as found in the first edition of the *Principia* and as modified by the second edition. The informal discussions will be ignored unless they furnish needed information not obtainable from the formal account.‡

I begin with a brief account of the theory.

2. *Brief Account of the Theory.* The theory of deduction is "the calculus of propositions" (p. 88); it is "the theory of how one proposition can be inferred from another" (p. 90). As a mathematical science, as "the most elementary part of mathematics" (p. 115), the theory consists of a certain set of primitive

---

* Presented to the Society, June 20, 1929, as part of a paper entitled *On Whitehead and Russell's theory of deduction.*

† Whitehead and Russell, *Principia Mathematica*, vol. 1, second edition, pp. 87–126. In later references to pages, this volume of the *Principia* will be understood.

‡ There are some discrepancies between the informal and the formal accounts of the theory.