A NOTE ON GEOMETRICAL FACTORIAL SERIES

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1. *Introduction.* This note is concerned with series of the form

$$(1) \qquad c_0 + \sum_{\nu=1}^{\infty} \frac{c_\nu}{(x-q)(x-q^2)\cdots(x-q^\nu)},$$

where q is such that $|q| < 1$. Series of this form are called geometrical factorial series. By comparing (1) with the descending power series

$$(2) \qquad c_0 + \sum_{\nu=1}^{\infty} c_\nu x^{-\nu}$$

it is shown that they have precisely the same points of convergence, and hence a number of fundamental properties of (1) follow from the known properties of the descending power series. In §3 a fundamental theorem concerning the representation of analytic functions by means of series of the form (1) is proved. In §4 precise formulas for the multiplication of two such series are obtained. The points $x = q^i$, $(i = 1, 2, 3, \cdots)$, will be called exceptional points for the series (1), and we agree to exclude such points from consideration. If we replace x by x^{-1}, the series (1) becomes

$$c_0 + \sum_{\nu=1}^{\infty} \frac{c_\nu x^\nu}{(1-qx)(1-q^2x)\cdots(1-q^\nu x)},$$

which, if it converges at all, converges for values of x in the neighborhood of the origin and defines an analytic function in such a region. Such series might be compared to the ascending power series. This discussion is presented with reference to the series (1) only because of convenience and simplicity of formulas.

2. *Comparison of* (1) *and* (2). Suppose x to be a point of convergence of (2). Let us write

$$u_\nu = c_\nu x^{-\nu}, \quad v_\nu = \frac{x^\nu}{(x-q)(x-q^2)\cdots(x-q^\nu)}.$$