PROJECTIVE INTEGRAL INVARIANTS ATTACHED TO THE TRAJECTORIES OF DIFFERENTIAL SYSTEMS*

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1. *Introduction.* In his book on integral invariants, E. Cartan† showed how to every integral invariant in the sense of Poincaré there corresponds a more general integral invariant (called *complete* by Cartan) in an associated manifold of one more dimension. It turns out, as Goursat‡ has more recently shown, that a complete integral invariant is a Poincaré integral invariant attached to the trajectories of an associated differential system.

Cartan, however, did not give necessary and sufficient conditions; neither did he go into the question of the existence of other integral invariants in the same associated manifold. The present paper is concerned with such problems. As in my previous contributions§ to the subject of integral invariants, the tensor methods and the language of group theory will be employed throughout the paper.

2. *Projective Transformations of Coordinates and Projective Tensors.* In this paragraph and throughout our whole paper we shall understand that *a Greek index can take on any of the values* $0, 1, 2, \cdots, n$, *while a Latin index can take on any of the values* $1, 2, \cdots, n$. The repetition of an index in a term will be used to denote summation with respect to that index over all its admissible values.

Let $\xi^i(x^1, \cdots, x^n)$ be a contravariant vector in an n-dimensional manifold $(x^1, x^2, \cdots, x^n)$. Consider an associated $(n+1)$-dimensional manifold with points having coordinates $(x^0, x^1, \cdots, x^n)$ and subject to the analytic transformations

$$(1)\qquad \mathfrak{G}: \begin{aligned} \bar{x}^i &= f^i(x^1, x^2, \cdots, x^n), \\ \bar{x}^0 &= x^0 + f^0(x^1, x^2, \cdots, x^n), \end{aligned} \qquad \left| \frac{\partial f^i}{\partial x^j} \right| \neq 0,$$

* Presented to the Society, November 29, 1930.

† E. Cartan, *Leçons sur les Invariants Intégraux*, 1922.

‡ E. Goursat, Comptes Rendus, vol. 174 (1922), pp. 1089–1091.

§ A. D. Michal, Transactions of this Society, vol. 29 (1927), pp. 612–646.