NEW EDITION OF OSGOOD ON FUNCTIONS

Lehrbuch der Funktionentheorie. By W. F. Osgood. Vol. I, fifth edition, 1928. 14+818. Vol. II, part 1, second edition, 1929. 7+307 pp. Leipzig and Berlin, Teubner.

The second edition of the first volume of this work was reviewed by E. B. Van Vleck in this Bulletin, (vol. 20 (1913), pp. 532–546). No reviews of the intervening editions have appeared in this journal. The first edition of the first half of the second volume was also reviewed by Professor Van Vleck, in this Bulletin (vol. 33 (1927), pp. 358–365). Both of these reviews are adequate and authoritative in giving an excellent idea of the plan of the work and of its chief characteristics. As a consequence we can limit ourselves in this review to the changes which have been made in these editions. In point of size, the fifth edition of the first volume contains some fifty odd pages more than the second. The added material is scattered through the volume, including changes in expressions, added exercises, added explanations, and new material.

The added material in the earlier chapters of the book include such matters as a separate section devoted to the definite (Cauchy) integral, its definition and properties, paragraphs on Abel's theorem on the continuity of a power series at a point of convergence on the circle of convergence, on iterated integrals, on the convergence of products of power series (it is puzzling why this was omitted in the early editions, when division and other arithmetic operations were included) and a section devoted to the Poincaré theta series. In the chapter on the logarithmic potential function the section on the motion of an incompressible fluid has been considerably expanded. The section covering the matter of isolated singularities of harmonic functions and the symmetry of Green's functions has been rearranged to advantage. Further, a paragraph giving the Arzelà theorem on compactness of a sequence of continuous functions has been added. The last chapter dealing with the uniformisation problem, aside from a number of minor changes, has been augmented by a section deriving, in detail, results concerning the analysis situs of the Riemann surfaces involved in the proof. Further, the final paragraphs, dealing with the uniformisation of an arbitrary analytic function, and the proof of the existence theorem for a many-valued function on an arbitrary region of definition, have been considerably revised. This has helped to make the subject matter more intelligible, the developments clearer, and have added materially to the usefulness of the volume, both as an introduction to the theory of functions, particularly the uniformisation problem, and as a work of reference.

Most of the exercises added to the later editions are of the thought provoking variety, frequently giving points of theory not covered in the text. One finds, for instance, as exercises, to prove that the sum of the residues of a rational function is zero, to discuss the many-valuedness of z^α, $(\alpha=a+ib)$, to prove that a monogenic analytic function can be at most denumerably infinitely many-valued, and so on. Putting questions of this character into problem form is certainly desirable, and could be carried to even further extremes.

If one were in a quibbling mood, one might take objection to the introduction of the definite integral on the basis of the most primitive form, of equal