EXTENSION OF A THEOREM OF MAZURKIEWICZ*

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S. Mazurkiewicz,† in answer to a question proposed by B. Knaster,‡ has shown that if A is a closed point set in E_n (euclidean space of n dimensions) which is homeomorphic with a subset of E_{n-1}, then all points of A are accessible from the complementary set, E_n-A. The question naturally arises, then, as to whether the points of A are *regularly* § accessible from E_n-A. It will be shown in the present paper that this is indeed the case.

We shall precede our proof by two theorems which, we believe, are themselves of fundamental importance. Following Mazurkiewicz' notation, we shall denote by $S_n(p, \rho)$ a spherical neighborhood of a point p of E_n with radius ρ; by $\phi(A)$, the subset of E_{n-1} that is homeomorphic with A; and if X is any subset of A, by $\phi(X)$ we denote that subset of $\phi(A)$ that corresponds to X under the homeomorphism between A and $\phi(A)$. Also, following the usual custom, if M is a point set, by $\overline{M}$ we shall denote the set M together with all of its limit points.

Evidently the proof given by Mazurkiewicz for his *Lemme* establishes the following more general lemma.

LEMMA 1. *Let P be a point of A, D a domain¶ containing P, and G a component of $D-A\cdot D$ such that $\overline{G}\supset P$. Then, if D_1 is a bounded domain such that $D_1\subset D$ and $D_1\supset P$, there is a component G_1 of $G\cdot D_1$ such that $P\subset \overline{G}_1$.*

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† *Sur un problème de M. Knaster*, Fundamenta Mathematicae, vol. 13 (1929), pp. 146–150.

‡ See Fundamenta Mathematicae, vol. 8 (1926), Problem 43, p. 376.

§ A point P of a point set M is said to be *regularly* accessible from a point set R of which P is a limit point provided that for every $\epsilon>0$ there exists a positive number δ such that if Q is a point of R whose distance from P is less than δ, then there is an arc from P to Q whose diameter is less than ϵ and which lies, except for P, wholly in R. See G. T. Whyburn, this Bulletin, vol. 34 (1928), p. 509.

¶ By *domain* we mean a connected open subset of the space under consider ation. The domain D may, of course, be E_n, in which case the component G of this lemma will necessarily exist, due to the invariance of dimensionality under analysis situs transformations.