LEMMA 3. *For every even* $p \geqq 2$,

$$A_p \leqq A_{p-1} + 1, \quad a_{p+1} \leqq a_p + 1; \tag{11}$$

$$a_{p-1} \leqq a_p + 3, \qquad A_p \leqq A_{p+1} + 3. \tag{12}$$

The proof of (11_1) is typical. By (9_2) and (9_3) with $p-1$ and p in place of p, $A_p \equiv A_{p-1}+1 \pmod 4$. Hence the contrary of (11_1) would imply $A_p = A_{p-1}+5+4v_1$, and consequently $b_p = b_{p-1} - 7 - 6v_1$, where $v_1 \geqq 0$. Hence, by $4A_p \geqq (b_p)^2$,

$$4(A_{p-1} + 5 + 4v_1) \geqq (b_{p-1} - 7 - 6v_1)^2,$$

contradicting (10_1) with $p-1$ in place of p, since

$$4(1 + 4v_1) \leqq 2(1 + 6v_1)(6 - b_{p-1}) + (6v_1 + 1)^2.$$

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GROUPS GENERATED BY TWO OPERATORS WHOSE SQUARES ARE INVARIANT

BY G. A. MILLER

It is well known that two operators of order two generate the dihedral group whose order is twice the order of the product of these operators. The groups that can be generated by two operators which have a common square are also well known. The groups considered in the present article are obviously a generalization of these two categories of well known groups. We shall represent their two generators by s and t. From the fact that s^2 and t^2 are invariant operators of the group G generated by s and t it results directly that

$$s^{-1}sts = t^{-1}stt = ts = (st)^{-1}s^2t^2,$$

$$s^{-1}tss = t^{-1}tst = st = (ts)^{-1}s^2t^2.$$

From these equations it follows that the abelian group H generated by s^2, t^2, and st is invariant under G and that its index under G cannot exceed 2.

A necessary and sufficient condition that H be identical with G is that G be abelian and can be generated by the product of two of its operators and the squares of these operators. It is not