ON A TYPE OF ILLUSORY THEOREM CONCERNING HIGHER INDETERMINATE EQUATIONS*

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1. *Introduction.* Let S be a system of equations, some or all of which need not be algebraic, finite or infinite in number, in a finite number μ of independent variables $x_1, \cdots, x_\mu$. If the total number of sets of positive (greater than zero) integral solutions $x_1 = x_1', \cdots, x_\mu = x_\mu'$ of S is finite, we shall refer to these sets as all the positive integral solutions of S. If S has an infinity of sets of positive integral solutions, we shall assume that a set of conditions C may be imposed on $x_1, \cdots, x_\mu$ such that S subject to C has only a finite number of sets of positive integral solutions, and we shall refer to these as all the positive integral solutions of S (so that reference to C may be omitted). To indicate the variables when it becomes necessary, we shall write $S \equiv S\ (x_1, \cdots, x_\mu)$.

Considering the generality of S, one would suspect that it is difficult to state any non-trivial theorem about all the positive integral solutions of S. Nevertheless several theorems concerning these solutions (the immaterial restriction to a finite number of equations being assumed) occur in the literature and have the appearance at first sight of being genuine theorems. The proof of the theorem in §2 reveals its true character; the generalization of this theorem in §3 scarcely needs a proof, while the further generalization in §4, devised for the occasion, betrays the nature of all such theorems. It is to be noticed that the theorems are much worse than trivialities; they direct us to prolix and unnecessary tentative calculations to settle questions whose answers are presupposed from the beginning.

2. *An Illusory Theorem.*† *If in all the positive integral solutions* $x_1, \cdots, x_\mu$ *of* $S(x_1, \cdots, x_\mu)$ *we set* $x_i = d_i\delta_i$, $(d_i, \delta_i$ *positive integers) in all possible ways, then the number of positive integral solutions* $y_1, \cdots, y_\mu$ *of* $S(y_1^2, \cdots, y_\mu^2)$ *is* $\Sigma\lambda(d_1 d_2 \cdots d_\mu)$,

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† Liouville, Journal de Mathématiques, (2), vol. 4 (1859), pp. 271–272; stated without proof; reported in Dickson's *History*, vol. 2, p. 700.