TWO TYPES OF CONNECTED SETS*

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1. *Definitions.* A set W will be said to be *widely connected* if it is connected† and every connected subset is everywhere dense in it.

A connected set W will be said to be *n-point connected*, where n is any given cardinal number, if there does not exist a subset of power n which disconnects W.

A connected point set (or a continuum) W of type T will be said to be a *perfect connected set* (or a *perfect continuum*) *of type* T if every connected subset (or every subcontinuum) of W is of type T. For example a *perfect one-point connected set* is a connected set, every connected subset of which is one-point connected. A *perfect indecomposable continuum* is a continuum every subcontinuum of which is indecomposable.‡

2. *An Example of a Widely Connected Set.* It will now be shown that under certain logical assumptions, including Zermelo's postulate,§ a widely connected set can exist.

THEOREM 1. *Any bounded indecomposable continuum M, lying in a euclidean space, contains a widely connected subset which is everywhere dense in M.*

Let (K) be the set whose elements are the composants¶ of M, these elements being contained but once in (K). It is known

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* Presented to the Society, October 25, 1930.

† By the notation $W = W_1 + W_2$ *separate* is meant that W is the sum of the two non-vacuous, mutually exclusive subsets W_1 and W_2 neither of which contains a limit point of the other. A set W is *connected* if there do not exist subsets W_1 and W_2 such that $W = W_1 + W_2$ separate. In this paper a single point will not be considered a connected set. By the notation W' will be meant the set W plus the limit points of W.

‡ See R. L. Wilder, *Characterizations of continuous curves that are perfectly continuous*, Proceedings of the National Academy of Sciences, vol. 15 (1929), pp. 614–621.

§ See Alonzo Church, *Alternatives to Zermelo's assumption*, Transactions of this Society, vol. 29 (1927), pp. 178–208.

¶ For definition and properties see Z. Janiszewski and C. Kuratowski, *Sur les continus indécomposables*, Fundamenta Mathematicae, vol. 1, pp. 217–222.