LINEAR FUNCTIONAL TRANSFORMATIONS IN GENERAL SPACES†

BY T. H. HILDEBRANDT

1. *Introduction.* An abstract theory of linear functional transformations has as guide linear transformations in a finite or denumerably infinite set of variables, linear integral transformations and equations associated with these. The desire to proceed symbolically and replace details by general procedure seems to be inherent in the situation. Pincherle‡ is perhaps one of the first great exponents, so that he even seems to have anticipated some of the famous results of integral equations by a number of years. E. H. Moore§ set himself the task of unifying the Fredholm theory of integral equations and algebraic equations in finitely and infinitely many variables, and has succeeded in setting up a system which indicates in a host of special cases a valid and elegant method of procedure analogous to the Fredholm integral equation theory. Volterra¶ has devised an elegant theory of linear integral and associated operations based on the notion of permutability or commutativity of operations.

The theory to which the main portion of this address is devoted has not been, in the main, presented as such in published form. It is, however, obvious that F. Riesz in his book entitled *Les Equations Linéaires à une Infinité d'Inconnus*‖ and his paper, *Lineare Funktionalgleichungen*†† has had in mind the generalizations treated here. As a consequence, what is given here is in the main hardly new, excepting that by presenting it from the point of view of a general basis, there is a gain in elegance and simplicity for the non-pathological results.

---

† An address delivered at the Summer Meeting of the Society at Providence, September 10, 1930, by invitation of the program committee.

‡ See *Notice sur les travaux*, Acta Mathematica, vol. 46 (1925), pp. 341–362, especially p. 347, and pp. 351–354; *L'Operazioni Distributive*, Bologna, 1901; Encyclopédie des Sciences Mathématiques, vol. II, 26 (1912).

§ See this Bulletin, vol. 28 (1912), pp. 334–362; Proceedings of the Cambridge International Congress, 1912, vol. I, pp. 230–255.

¶ See for instance *Leçons sur les Fonctions des Lignes*, Paris, 1913, Chap. 9, etc.

‖ Paris, 1913; this book will be cited in what follows as Riesz, *Inf. Inc.*

†† Acta Mathematica, vol. 41 (1917), pp. 75–88; this paper will be cited in what follows as Riesz, *LFG.*