

GRAUSTEIN ON GEOMETRY

Introduction to Higher Geometry. By William C. Graustein. New York, The Macmillan Company, 1930. xv+486 pp., 126 figures.

During the last few years a number of excellent texts have appeared in English on analytic geometry of space, all of which either presupposed or treated in a brief and sketchy manner the foundations on which the theory was developed. And in the same time a number of synthetic projective geometries have been issued which adequately cover the field from that point of view.

It is therefore with particular pleasure that geometers welcome the volume under review. While somewhat belated as compared with the other books mentioned, it can now enjoy the double role of both cause and effect in the present re-awakening of interest in algebraic geometry, a field which requires a wider application of various mathematical disciplines than any other for its proper introduction.

For a book of this nature, three questions at once arise: what is its scope, what are its presupposed premises, and what are its methods? The first and second can be answered in a word, while the third question will be considered more in detail. The scope is to fix the foundations for the study of analytic projective geometry, by establishing the coordinate systems in points, lines, and planes, and applying them to an extensive study of entities defined by linear and by quadratic equations, together with a few equations of higher degree. The field is sometimes the real continuum, sometimes complex, the distinction between the two being sharply drawn as occasion arises.

A knowledge of cartesian analytic geometry of two and three dimensions, of elementary properties of determinants and of the theory of equations, and of the processes of the calculus are presupposed, in so far as these subjects are developed in a first college course in each. In fact a very elementary knowledge of determinants is sufficient.

The book begins with a review of determinants and the solution of a system of simultaneous linear equations, bringing in the ideas of a matrix and of linear dependence from a purely algebraic point of view. Then a synthetic definition of projection, of vanishing points and lines, the theorem of perspective triangles and the principle of duality. Homogeneous coordinates are introduced as ratios, first for the plane, from cartesian coordinates, and the line at infinity is carefully adjoined to the euclidean plane. The geometric meaning of linear dependence is now introduced, and an analytic proof of the plane perspective triangle theorem is given. Harmonic section is approached from the metric side, interpreted in terms of positive and negative line segments on a straight line. Line coordinates are defined as the coefficients in the equation of a straight line, and the earlier definition of duality is justified by the identical algebra in point and line coordinates. Cross ratio is introduced metrically, but is at once shown to be invariant. Linear transformations are approached by rigid motions, which are shown to form a group; then follows a systematic treatment of one-dimensional linear transformations and a less extensive one in two dimensions.