KELLOGG ON POTENTIAL

Foundations of Potential Theory. By O. D. Kellogg. Berlin, Springer, 1929. ix+384 pp.

The book before us is modern in its field of interest, and is nevertheless based entirely on the kind of mathematics with which the advanced undergraduate, or young graduate, is familiar; hence the American student has now an opportunity, if he will substitute a European for an American habit, and buy a book, to be well grounded in the fundamental subject of potential theory. He will thus possess knowledge which not only has contact with much of the past development of the theory of partial differential equations and its applications to "classical" mathematical physics and is most important in its relations to functions of a complex variable, but also continues to introduce him to the ever new in mathematical problems. Suffice it here to mention in the last direction the problem of Plateau, which lately, by the acquisition of new existence theorems, has opened up large domains for investigation.

The range of subject is suggested by the various chapter headings: The force of gravity; Fields of force; The potential; The divergence theorem; Properties of Newtonian potentials at points of free space; Properties of Newtonian potentials at points occupied by masses; Potentials as solutions of Laplace's equation, electrostatics; Harmonic functions; Electric images, Green's function; Sequences of harmonic functions; Fundamental existence theorems; The logarithmic potential. Velocity fields and the equation of continuity are treated with fields of force, the Heine-Borel theorem occurs in connection with the divergence theorem, developments valid at great distances are made essential for the properties of potentials, ellipsoidal coordinates and the potential of the homogeneous ellipsoid are given in connection with potentials as solutions of Laplace's equation (see Jeans's ingenious generalization of such formulas in the problem of equilibrium of rotating fluids!), conformal mapping is treated under the logarithmic potential, and so on. The final section is devoted to the explicit formulas required for the mapping of polygons.

In a subject of this character it cannot be hoped to make a complete exposition. Beyond the fundamentals, the author will limit himself somewhat narrowly, knowing that although he must omit direct treatment of many subjects he will cross them in the line of march in such a way that the student may return to them later. Thus in the chapter on *Sequences of harmonic functions*, Professor Kellogg is able to devote an incidental four pages to development in spherical harmonics. The main line of march here, indeed, throughout the last third of the book, is towards the exploration of harmonic functions defined in arbitrary regions and the Dirichlet problem. This problem, in which the boundary values are assigned continuously, is a central problem for potential theory.

In order to make sound the treatment of integrals over regions and surfaces it is necessary to discuss the nature of the regions and their bounding surfaces with considerable care and detail. Green's and Stokes' theorems are estab-