A NOTE CONCERNING CACTOIDS*

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A *cactoid*‡ M is a bounded continuous curve lying in space of three dimensions and such that (a) every maximal cyclic curve§ of M is a simple closed surface and (b) no point of M lies in a bounded complementary domain of any subcontinuum of M. There exists a bounded acyclic|| continuous curve C such that every bounded acyclic continuous curve is homeomorphic with a subset of C. Now Whyburn has shown¶ that with respect to its cyclic elements every continuous curve is acyclic. Moreover the cyclic elements of a cactoid are either points or topological spheres. Thus this question naturally arises: Does there exist a cactoid C such that every cactoid is homeomorphic with a subset of C? The object of the present paper is to answer this question negatively.

THEOREM 1. *There does not exist a cactoid C such that every cactoid is homeomorphic with a subset of C.*

PROOF. Let g be any infinite set of distinct positive integers $d_1, d_2, d_3, \cdots$. Let K denote a non-dense perfect point set on the interval $0 \leqq x \leqq 1$ containing the end points of this interval. The complementary segments of K can be labeled

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‡ See R. L. Moore, *Concerning upper semi-continuous collections*, Monatshefte für Mathematik und Physik, vol. 36 (1929), p. 81.

§ For a definition of this term, and of the term *cyclic element*, see G. T. Whyburn, *Concerning the structure of a continuous curve*, American Journal of Mathematics, vol. 50 (1928), p. 167.

|| See T. Wazewski, *Sur les courbes de Jordan ne renfermant aucune courbe simple fermée de Jordan*, Annales de la Société Polonaise de Mathématique, vol. 2 (1923), p. 57. See also Menger, *Über allgemeinen Kurventheorie*, Fundamenta Mathematicae, vol. 10 (1926), p. 108. In his paper *On continua which are disconnected by the omission of any point and some related problems*, Monatshefte für Mathematik und Physik, vol. 35 (1929), p. 136, W. L. Ayres extends this result to unbounded acyclic continuous curves. An *acyclic* continuous curve is one which contains no simple closed curve.

¶ Loc. cit., pp. 167–194.