As *dual* partial correlation coefficient we might take the invariant numbers

$$\rho_{jk} = \frac{\sigma_{jk}}{(\sigma_{jj})^{1/2}(\sigma_{kk})^{1/2}}.$$

All invariant functions of the quantities σ_{jk} are also functions of the quantities ρ_{jk} only. But the quantities ρ_{jk} have not the simple relation to the regression hyperplanes.

In problems of type B in more variables the symmetry axes of the correlation quadric come into consideration.

In problems of type C the regression planes and the correlation coefficients lose their sense, but not the symmetry axes. Here the theory becomes the well known theory of quadratic matrices under orthogonal substitutions with the unessential modification that similarity transformations are also permitted.

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NOTE ON THE EXISTENCE OF A POSITIVE FUNCTION ORTHOGONAL TO A GIVEN SET OF FUNCTIONS*

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Let the finite set of functions:

$$\{f_j(x)\}: \qquad f_1(x), f_2(x), \cdots, f_m(x)$$

be continuous and linearly independent on the closed interval X, $(a \leqq x \leqq b)$. With reference to this set of functions, L. L. Dines‡ has shown the equivalence of the following properties:

(A) *Every linear combination of the functions changes sign on X.*

(B) *There exists a positive continuous function orthogonal to each function of the set on X.*

A sufficient condition for the set $\{f_j(x)\}$ to have properties (A) and (B) has also been given by Dines.§ It is in a form

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‡ *A theorem on orthogonal functions with an application to integral inequalities*, Transactions of this Society, vol. 30 (1928), pp. 425–438.

§ *On completely signed sets of functions*, Annals of Mathematics, vol. 28 (1926), pp. 393–395.