ON CERTAIN PROBLEMS OF APPROXIMATION IN THE COMPLEX DOMAIN*

BY DUNHAM JACKSON

When the paper with the above title was presented to the Society, I expected to publish it within a short time, and put on record at the moment only a brief abstract giving no very specific indication as to the results obtained. But the publication was delayed, and meanwhile J. L. Walsh,† working quite independently, has treated problems of similar character by a different method, and has given a discussion which is in various ways considerably more thorough and comprehensive than that which I had contemplated, besides extending to other problems which I had not dealt with at all. Some of my conclusions nevertheless remain outside the scope of Walsh's article, and the purpose of the following paragraphs is to give an account of these results, with repetition of those stated by Walsh only so far as is necessary in order to exhibit the working of the method. The reader is referred to Walsh's paper also for detailed citations of the literature.

The problem of the present note is that of the convergence of a sequence of approximating polynomials for a function of a complex variable, the approximating polynomials being chosen so as to minimize an integral containing a power of the absolute value of the error. The method is an adaptation of one which has been applied to problems in the approximation of real functions, depending on Bernstein's theorem on the derivative of a real polynomial or of a trigonometric sum. A corresponding theorem for polynomials in the complex domain reads as follows:‡

If $P_n(z)$ is a polynomial of the nth degree such that

$$|P_n(z)| \leqq L \qquad \text{for} \qquad |z - z_0| \leqq R,$$

* Presented to the Society, December 29, 1927.

† *On the overconvergence of sequences of polynomials of best approximation*, Transactions of this Society, vol. 32 (1930), pp. 794–816.

‡ S. Bernstein, *Leçons sur les Propriétés Extrémales et la Meilleure Approximation des Fonctions Analytiques d'une Variable Réelle*, Paris, Gauthier-Villars, 1926, pp. 44–45.