

CONCERNING A SET OF AXIOMS FOR THE SEMI-
QUADRATIC GEOMETRY OF A THREE-SPACE*

BY J. L. DORROH

In his paper *Sets of metrical hypotheses for geometry*,† R. L. Moore raises the question whether the set O of order axioms and the set C of congruence axioms employed therein, together with M , the proposition that every segment has a mid-point, and P_2 , a form of the parallel postulate, are sufficient to give the semi-quadratic geometry of a three-space. At the same time, he states that this question may be answered in the affirmative if it can be proved on the basis of O , C , and M that all right angles in space are congruent to each other. In the present paper it will be shown that O and C are sufficient to require that all right angles in space be congruent to each other.

It is a result of a recent paper‡ of the present author that the theorems of sections 1, 2, 3, and 4 of M.H. are consequences of O and C . Theorems from these sections of M.H. will be quoted without further mention of this justification of their use.

THEOREM 1. *If A, B, C, D are four non-coplanar points such that $\sphericalangle ABD$ is a right angle§ and $\sphericalangle CBD$ is a right angle, and E is any point distinct from B and in the plane ABC , then $\sphericalangle EBD$ is a right angle.*

PROOF. If E is a point of the line AB , or of the line CB , then, by hypothesis, $\sphericalangle EBD$ is a right angle.

Suppose, then, that E belongs to the plane ABC , is distinct from B , and belongs neither to the line AB nor to the line BC . Let C' denote a point such that CBC' . It follows by a corollary

* Presented to the Society, September 6, 1928.

† Transactions of this Society, vol. 9 (1908), pp. 487-512. The notation M. H. will be used to designate this paper. Similarly, S. A. will be used to denote O. Veblen's paper, *A system of axioms for geometry*, *ibid.*, vol. 5 (1904), pp. 343-384.

‡ *Concerning a set of metrical hypotheses for geometry*, *Annals of Mathematics*, (2), vol. 29 (1928), pp. 229-231.

§ See Definition 7 of M. H., §3.