The conditions of the theorem are necessary by the results of §3 and the corollary of §8. We may show that the conditions are sufficient by an argument following closely that used in proving the sufficiency of the condition in §6.

The first example of §7 shows that the conditions of the theorem are not sufficient if we do not specify that M is locally connected.

THE UNIVERSITY OF MICHIGAN

ON THE GENERALIZATION OF TRIGONOMETRIC IDENTITIES IN ARITHMETICAL PARAPHRASING*

BY H. T. ENGSTROM†

1. *Introduction.* Identities of the type

$$\sum_{s=1}^{m}\alpha_s \sin a_s x \equiv \sum_{t=1}^{n}\beta_t \sin b_t x, \tag{1}$$

where α_s, a_s, β_t, b_t are rational integers, arise in the comparison of like powers of the modulus when an elliptic function is represented in more than one way by trigonometric series. The following theorem is used in obtaining arithmetical results from such identities.

THEOREM 1. *If $g(x)$ is an arbitrary, single-valued, odd function, defined for $x=a_s$, $s=1, 2, \cdots, m$, and $x=b_t$, $t=1, 2, \cdots, n$, then* (1) *implies*

$$\sum_{s=1}^{m}\alpha_s g(a_s) = \sum_{t=1}^{n}\beta_t g(b_t). \tag{2}$$

Similarly, for cosines, we have the following statement.

THEOREM 2. *If $f(x)$ is an arbitrary, single-valued, even function, defined for $x=a_s$, $s=1, 2, \cdots, m$, and $x=b_t$, $t=1, 2, \cdots, n$, then*

$$\sum_{s=1}^{m}\alpha_s \cos a_s x \equiv \sum_{t=1}^{n}\beta_t \cos b_t x \tag{3}$$

---

* Presented to the Society, April 5, 1930.

† National Research Fellow, California Institute of Technology.