ON THE DENSITY OF THE CUT POINTS AND END POINTS OF A CONTINUUM*

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1. *Introduction.* We consider a connected, compact, metric space M which we refer to as the continuum M. The space M is said to be *locally connected* if for each point p of M and each $\epsilon>0$ there exists a $\delta>0$ and a connected set N such that $S(p, \delta) \subset N \subset S(p, \epsilon)$. A point p is said to be a *cut point* of M if $M-p$ is not connected. A point p is said to be an *end point* of M if for each $\epsilon>0$ there exists a neighborhood U_p such that $U_p \subset S(p, \epsilon)$ and $B(U_p)=\overline{U}_p-U_p$ is a single point.† From this definition it is seen that every end point is a limit point of the cut points of M. Hence whenever the end points are dense in M, the cut points are also dense. This relation is not true conversely, but a study of some examples leads one to the conclusion that there exist some fundamental relations between the density of the cut points and the density of the end points. In this note we shall investigate some of these relations.

2. *Notation.* Let K and E denote respectively the set of all cut points and end points of M. Let K^2 denote the set of all cut points which are of Urysohn-Menger order 2 in M and let ${}^2K$ denote the set of all cut points which are of order >2. Capitals will denote sets of points, lower case letters single points. $S(p, \epsilon)$ denotes the set of all points whose distance from p is less than ϵ. The symbol $\rho(x, y)$ denotes the distance from x to y; $\rho(X, Y)$ denotes the greatest lower bound of the numbers $\rho(x, y)$ where $x\varepsilon X$ and $x\varepsilon Y$. The notations $x\varepsilon X$ and x non-$\varepsilon\, X$ mean "x is a point of the set X" and "x is not a point of X" respectively. The symbol $d(X)$ denotes the diameter of X, that is, the least upper bound of all numbers $\rho(x, y)$ where $x\varepsilon X$ and $y\varepsilon X$.

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† We use end point here in the Urysohn-Menger sense. See P. Urysohn, Comptes Rendus, vol. 175 (1922), pp. 481–483; and K. Menger, Mathematische Annalen, vol. 95 (1925), pp. 277–306. For other senses in which the term has been used, see H. M. Gehman, *Concerning end points of continuous curves and other continua*, Transactions of this Society, vol. 30 (1928), pp. 63–84.