ON CANONICAL FORMS OF DIFFERENTIAL EQUATIONS*

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1. *Introduction.* The projective differential geometry of surfaces has been greatly enriched by the introduction of Fubini's projective normal into the theory. This projective normal however fails to exist for ruled surfaces. In this paper we propose a class of congruences covariantly related to the surface, each congruence being suitable as a substitute for Fubini's projective normal congruence. As an application we give an interesting class of congruences covariantly related to a scroll surface and conjugate to the surface.

If the asymptotic parameters are parametric, the four homogeneous coordinates $y^{(1)}$, $y^{(2)}$, $y^{(3)}$, $y^{(4)}$ of a general point on the surface S_y are solutions of differential equations in the Wilczynski semicanonical form:†

$$(1)\qquad \left\{\begin{aligned} y_{uu} + 2ay_u + 2by_v + cy &= 0,\\ y_{vv} + 2a'y_u + 2b'y_v + c'y &= 0.\end{aligned}\right.$$

One of the integrability conditions of system (1) is

$$(2)\qquad a_v = b'_u\,.$$

If the transformations

$$(3)\qquad y = \lambda\bar{y}$$

and

$$(4)\qquad \bar{u} = U(u),\quad \bar{v} = V(v)$$

are performed on system (1), the new coefficients are respectively

$$(5)\qquad \left\{\begin{aligned} &\bar{a} = a + \frac{\lambda_u}{\lambda}, \quad \bar{b} = b, \qquad \bar{c} = \frac{1}{\lambda}(\lambda_{uu} + 2a\lambda_u + 2b\lambda_v + c\lambda),\\ &\bar{a}' = a', \qquad \bar{b}' = b' + \frac{\lambda_v}{\lambda}, \quad \bar{c}' = \frac{1}{\lambda}(\lambda_{vv} + 2a'\lambda_u + 2b'\lambda_v + c'\lambda),\end{aligned}\right.$$

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† E. J. Wilczynski, *First memoir*, Transactions of this Society, vol. 8 (1907), pp. 233–260.