NEW CRITERIA ASSOCIATED WITH FERMAT'S LAST THEOREM*

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Furtwängler has obtained* by means of Eisenstein's law of reciprocity for residues of pth powers, p an odd prime, certain criteria in connection with the solution of the equation

$$x^p + y^p + z^p = 0, \tag{1}$$

where x, y, z are relatively prime rational integers, and these criteria involve the rational factors of x, y, z, $y-z$, $z-x$, $x-y$.

It is the object of the present article to employ the same method to derive similar criteria for the factors of

$$x^2 - yz,\ y^2 - zx,\ z^2 - xy,\ x^2 + yz,\ y^2 + zx,\ z^2 + xy.$$

THEOREM 1. *If x, y, z satisfy equation* (1), *$yz+zx+xy$ is prime to p, and r is any factor of x^2-yz, then* $r^{p-1}\equiv 1 \pmod{p^2}$.

PROOF. Let α be a pth root of unity. We have from the identity

$$x(x + y\alpha) - y(z + x\alpha) = x^2 - yz,$$

the equation in pth power characters

$$\left\{\frac{x(x + y\alpha)}{r}\right\} = \left\{\frac{y(z + x\alpha)}{r}\right\}$$

or, since

$$\left\{\frac{x}{r}\right\} = \left\{\frac{y}{r}\right\} = 1,$$

x and y being rational, we have

$$\left\{\frac{x + y\alpha}{r}\right\} = \left\{\frac{z + x\alpha}{r}\right\}. \tag{2}$$

* Presented to the Society, August 29, 1929.

† Wiener Sitzungsberichte, vol. 121, IIa (1912), pp. 589–592.