THE RATIONALITY OF CERTAIN CONTINUOUS CURVES

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1. *Introduction.* In this paper we establish the following theorem.

THEOREM. *Every plane continuous curve M every subcontinuum of which is a continuous curve is a rational curve.*†

We first state two lemmas, both of which are very easy to prove.

LEMMA 1. *In order that any continuous curve M should be a rational curve it is necessary and sufficient that every maximal cyclic curve of M be a rational curve.*

LEMMA 2. *If P is any point of a plane continuous curve M, every subcontinuum of which is a continuous curve, then there exists a continuous curve M^*, also every subcontinuum of which is a continuous curve, which contains M and such that P is on the boundary of no complementary domain of M^*.*

The necessity of the condition in Lemma 1 is obvious. The sufficiency is established by the following steps: (a) a continuum M is a rational curve if and only if every two points A and B of M can be separated in M by a countable subset of M. (Menger, loc. cit., proves this where A and B are closed sets; a simple application of the Lindelöf Theorem suffices to prove the part not included in Menger's theorem); (b) if two points A and B of a continuous curve M lying together in a maximal cyclic curve C of M are separated in C by a closed subset K of C, then K also separates A and B in M, (because if not, then A and B lie together in a component H of $M-K$, and since $H \cdot C$

† A continuum M is said to be a rational curve if each point of M is contained in arbitrarily small neighborhoods in M with countable boundaries (see K. Menger, *Grundzüge einer Theorie der Kurven*, Mathematische Annalen, vol. 95 (1925), p. 277); or, in other words, for each point P of M and each $\epsilon>0$, P can be ϵ-separated in M by a countable subset of M (see P. Urysohn, *Mémoire sur les multiplicités Cantoriennes*, II Verhandelingen der Akademie Amsterdam, vol. 13, No. 4).