SHORTER NOTICES

Einführung in die Algebra. By Otto Haupt. Vols. I–II. Leipzig, Akademische Verlagsgesellschaft, 1929. 663 pp.

The domain of algebra has in recent years been enriched with a great number of German textbooks of high quality. I need only mention the books by Perron, Fricke, Hasse, and Bieberbach's edition of Bauer's algebra. The new algebra by Haupt might therefore at first glance seem somewhat superfluous. A closer study, however, reveals that it as an unusually successful attempt to present the foundations of modern abstract (commutative) algebra within the scope of a textbook.

The book is mainly based on the fundamental ideas of Steinitz set forth in his paper *Algebraische Theorie der Körper* (Journal für Mathematik, vol. 137 (1910), pp. 167–309). It contains numerous references to newer literature and several investigations are here for the first time presented in a textbook. The lucidity of its style makes it a convenient and attractive introduction to some very important parts of modern algebra.

The book starts with a discussion of the axioms of algebra, and then rings, domains of integrity and fields are introduced and the existence of quotient-fields is proved for all domains of integrity. In the third chapter the foundations of group theory are laid down; then follows an investigation of the axioms of divisibility, Euclid's algorithm, and unique decomposition in prime factors. Furthermore the classes of remainders are analysed and the abstract fields are classified according to their *characteristic* as done by Steinitz.

The next chapters in volume I are not very different from the representation of these matters in ordinary textbooks. The only difference is that the coefficients are always elements of an arbitrary abstract field, and that the fields of characteristic $p>0$ in certain cases may cause exceptions. The headings are: Transcendental adjunctions (that is, fields of rational functions of one or more variables), symmetric functions, linear equations, divisibility of polynomials in fields and domains of integrity. An interesting paragraph deals with the determination of the prime-function decomposition in a finite number of operations. The method applied is a direct generalization of Weber's method for algebraic fields.

The existence of abstract fields in which a given polynomial is equal to a product of linear factors is then proved; I remark, en passant, that the proof of this theorem is considerably simpler than the proof for the ordinary fundamental theorem of algebra, which states that *every* polynomial with coefficients in the complex field is equal to a product of linear factors in the same field.

All prime-functions can be divided into two classes: An irreducible polynomial belongs to the first class, when all its roots are different; if not, it belongs to the second class. A field in which all polynomials are of the first class is called *perfect.* The complete Galois theory can be extended to these fields. A field can only then be imperfect, when it has a characteristic $p>0$ and there exists an element a such that $\sqrt[p]{a}$ is not contained in the field.