(15), (16) between operations in the I-ring, and in IF, and the correspondence between IF and a ring of an abstract field, the theorem is proved.

The element $X(x)=\sum\xi(n)x^n$ of the I-ring is now defined to be *regular* or *irregular* according as $X'(x)=\sum\xi'(n)x^n$, where $\xi\xi'=\eta$, is or is not in the I-ring.

Let $A(x)=\sum\alpha(n)x^n$ be any element of the I-ring, and $B(x)=\sum\beta(n)x^n$ any regular element of the I-ring. Write $B'(x)=\sum\beta'(n)x^n$, where $\beta\beta'=\eta$. Then the *I-quotient* $A(x)/B(x)$ of $A(x)$ by $B(x)$ is defined by

$$A(x)/B(x) = A(x)B'(x); \tag{19}$$

$B'(x)$ is called the *I-reciprocal* of $B(x)$, and we write $B'(x) = U(x)/B(x)$. Combining (18), (19), we have the following theorem.

(20) THEOREM. *The set of all elements of the I-ring is an irregular field, say the I-field, in which division is as in* (19) *and the remaining fundamental operations as in* (18); *the irregular elements of the I-field are those of the I-ring.*

CALIFORNIA INSTITUTE OF TECHNOLOGY

ON TRI-RHAMPHOIDAL AND BI-OSCNODAL QUINTIC CURVES

BY HAROLD HILTON

In a recent paper,* T. R. Hollcroft says "For example, a quintic may have three rhamphoid cusps or two tacnode-cusps."

Now it is true that there is just one projectively distinct quintic with three rhamphoid cusps (or two, if we confine ourselves to real projections), namely

$$x:y:z = t^2(t - \tfrac{3}{2} - \tfrac{1}{2}\sqrt{5}) : t^2(t-1)^2(t - \tfrac{1}{2} + \tfrac{1}{2}\sqrt{5}) : (t-1)^2(t + \tfrac{1}{2} - \tfrac{1}{2}\sqrt{5}).$$

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* This Bulletin, vol. 35 (1929), p. 847.