IN- AND CIRCUMSCRIBED SETS OF PLANES TO SPACE CURVES*

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1. *Introduction.* The problem dealing with in- and circumscribed polygons to plane curves has been studied extensively by Durège,† Sylvester,‡ Story,§ Cayley,|| and others. The purpose of this paper is to investigate analogous problems for certain curves in 3-space and for the cuspidal space curve of order $n+1$ in n-space. A construction similar to that used for the in-and-circumscribed polygon, using, instead of tangent lines, hyperplanes having contact of order $n-1$, gives what may be called an in- and circumscribed set of planes to the n-space curve.

2. *Rational Quartic Curves in* 3-*Space.* The quartic curves are the curves of lowest order that need be considered since the osculating plane to a twisted cubic does not intersect the cubic again.

Let t_k be the parameter of the point at which the $(k-1)$st osculating plane intersects the quartic and at which the kth osculating plane has contact with the curve. It is evident that the necessary condition for an in- and circumscribed set of n planes is that $t_{n+1}=t_1$.¶

THEOREM I. *There are no in-and-circumscribed sets of osculating planes to the cuspidal quartic.*

Taking the equations of the cuspidal quartic in the form $x:y:z:w=t^4:t^3:t^2:1$, it is found that $t_{n+1}=(-1)^n t_1/3^n$. Hence the only solution of the equation $t_{n+1}=t_1$, in this case, is $t_1=0$, which gives no in- and circumscribed sets.

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† Durège, Mathematische Annalen, vol. 1 (1869), pp. 509–532.

‡ Sylvester, American Journal of Mathematics, vol. 2 (1879), pp. 381–387.

§ Story, American Journal of Mathematics, vol. 3 (1880), pp. 379–380.

|| Cayley, Collected Works, vol. 8, p. 212.

¶ In all cases only those sets that consist of two or more distinct planes are counted.