# A DEFINITION OF AN UNKNOTTED SIMPLE CLOSED CURVE*

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Mazurkiewicz and Strasziewicz have given a definition of interlacability.† That this definition can be altered to apply to a single curve so as to test for a knot in the curve will be shown in what follows.

Consider a three-dimensional, euclidean space. If for each point of the interval $t_1 \leqq t \leqq t_2$ we let the continuous function $f(t)$ define a point in the space, the set of points so formed will be a continuous curve. If $f(t_1) = f(t_2)$ the curve is a closed curve. If $f(t') = f(t'')$ implies that either $t' = t_1$, $t'' = t_2$ or $t' = t_2$, $t'' = t_1$ or $t' = t''$, then the curve is a simple closed curve and will be denoted by the symbol $[f(t); t_1, t_2]$.

DEFINITION. *If* $[f(t); t_1, t_2]$ *is a simple closed curve and if a uniformly continuous function* $f(t, \lambda)$, *where* $t_1 \leqq t \leqq t_2$, $0 \leqq \lambda \leqq 1$, *can be found having the following properties*:

$$f(t, 1) = f(t); \tag{1}$$

$$f(t, 0) = f_0, \quad \textit{a constant}; \tag{2}$$

$$f(t', \lambda') = f(t'', \lambda''), \tag{3}$$

*if and only if* (i) $\lambda' = \lambda'' = 0$, *or* (ii) $\lambda' = \lambda'' \neq 0$, *and one of the following hold*: (a) $t' = t''$, *or* (b) $t' = t_1$, $t'' = t_2$, *or* (c) $t' = t_2$, $t'' = t_1$; *then the simple closed curve* $[f(t); t_1, t_2]$ *is unknotted.*

It must now be shown that the ordinary properties of knots are impossible for curves satisfying this definition; that is, no knotted curve can be a subset of a set which is in 1–1 continuous correspondence with a plane and every unknotted curve can be exhibited as a subset of such a set.

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† Mazurkiewicz and Strasziewicz, *Sur les coupures de l'espace*, Fundamenta Mathematicae, vol. 9, p. 205.