PROBABILITY AS EXPRESSED BY ASYMPTOTIC LIMITS OF PENCILS OF SEQUENCES*

BY E. L. DODD

1. *Introduction.* From time to time certain conceptions and theorems of mathematics need a restatement. Especially true is this in a subject like probability throughout which is woven so much non-mathematical material. This paper, then, is an attempt to depict probabilities, initially discrete, by the use of aggregates of sequences, in language strictly mathematical, but as simple as possible. The treatment will resemble not so much that of von Mises,† who uses single sequences, as that of Borel,‡ Lomnicki,§ and Steinhaus.|| But it will differ from the latter two in avoiding the concept of measure, and indeed from all in the approach to the subject. The asymptotic limit introduced in this paper involves a notion resembling that involved in the phrase *almost everywhere.*

In pure mathematics, the word *probability* may be taken¶ to signify simply the ratio of the number of objects in a subset to the number in the set, so long as discrete or arithmetic probability is being considered. It is, indeed, as far outside the field of mathematics to determine whether two events are equally likely as to determine whether two bodies have the same mass. Even in the applications, the role of pure mathematics is merely to count expeditiously the elements of sets and subsets, or more generally to determine certain measures of sets, which are believed by competent judges to depict adequately situations in the external world. It is generally believed, for example,

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† *Grundlagen der Wahrscheinlichkeitsrechnung*, Mathematische Zeitschrift, vol. 5 (1919), pp. 52–99; see especially pp. 55–57.

‡ *Les probabilités dénombrables et leurs applications arithmétiques*, Rendiconti di Palermo, vol. 27 (1909), pp. 247–271.

§ *Nouveaux fondements du calcul des probabilités*, Fundamenta Mathematicae, vol. 4 (1923), pp. 34–71.

|| *Les probabilités dénombrables et leur rapport à la théorie de la mesure*, Fundamenta Mathematicae, vol. 4 (1923), pp. 286–310.

¶ See Lomnicki, loc. cit., p. 37.