ON THE REPRESENTATION OF ANALYTIC FUNCTIONS OF SEVERAL VARIABLES AS INFINITE PRODUCTS*

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1. *Introduction.* In a paper by J. F. Ritt soon to appear in the Mathematische Zeitschrift, he proves that any function $f(z)$, analytic, and equal to unity at $z=0$, can be represented in one and only one way as an infinite product $\prod_1^\infty(1+c_nz^n)$, which converges absolutely for $|z|\leqq 1/(6R)$, where R is the least upper bound of the. infinite sequence $|b_1|$, $|b_2|^{1/2}, \cdots$, $|b_k|^{1/k}, \cdots$ and b_k is the coefficient of z^{k-1} in the Taylor expansion of $f'(z)/f(z)$.

The object of this paper is to extend this result to functions of two variables. The first part will be concerned with a demonstration that an analytic function $f(x, y)=1+\sum b_{mn}x^my^n$ can be uniquely represented as an absolutely convergent infinite product $\prod(1+a_{mn}x^my^n)$ with constant a's. The second part will consider the representation of $f(x, y)$ in the form $\prod_1^\infty(1+P_n)$ where P_n is a homogeneous polynomial in x and y of degree n. Although the proof in each case is carried out for two variables, it will be evident how to extend it to functions of any number of variables. It should be noted, however, that the analytic functions considered in this paper constitute a restricted class of such functions, namely, those which equal unity at the origin. For one variable the corresponding assumption is not an essentially restrictive one.

2. THEOREM 1. *If $f(x, y)$ is analytic and equal to unity at $(0, 0)$, then in the neighborhood of $(0, 0)$ it admits a unique representation as an absolutely convergent infinite product*

$$f(x,y) = \prod_{m,n}(1 + a_{mn}x^my^n)$$

with constant a_{mn}.

Let the Taylor's expansion of $f(x, y)$ at $(0, 0)$ be $1+\sum b_{mn}x^my^n$. Since $f(0, 0)=1$ there is a neighborhood about $(0, 0)$ for which

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