INVERSE CORRESPONDENCES IN AUTOMORPHISMS OF ABELIAN GROUPS*

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The totality of the operators which correspond to their inverses in an automorphism of an abelian group G obviously constitutes a subgroup of G. It is well known that G cannot contain any characteristic operator besides the identity unless this operator is of order 2, and that it cannot contain more than one characteristic operator of this order. Hence it results that whenever G contains any characteristic operator besides the identity this operator must appear in every subgroup which is composed of all the operators of G which correspond to their inverses in a given automorphism of G. We proceed to prove that a necessary and sufficient condition that an automorphism of G can be established such that all the operators of a given subgroup H correspond to their inverses, while no other operator of G satisfies this condition, is that H involves all the characteristic operators of G. In particular, when G involves no characteristic operator besides the identity, it is possible to establish an automorphism of G such that all the operators of an arbitrary subgroup correspond to their inverses, while no other operator of G has this property.

When G is of order p^m, p being a prime number, and of type $(1, 1, 1, \cdots)$ it is well known that all of its operators besides the identity can be transformed cyclically and that none of its subgroups of order p is transformed into itself under such a transformation whenever $m>1$. Moreover, when all the operators of G are arranged in co-sets with respect to H, where each of the operators of H corresponds to its inverse, then either all the operators of a co-set correspond to their inverses in the same automorphism or none of these operators has this property. Hence the theorem under consideration is obviously true when G is of order p^m and of type $(1, 1, 1, \cdots)$. Moreover, it results from the fact that an abelian group contains only one

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