NOTES ON THE RATIONAL PLANE OSCNODAL QUARTIC CURVE*

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1. *Introduction.* In a recent paper†, which discussed the covariants of the curve R_2^4 with an oscnode, I made the statement that the quartic $\overline{12}^2(1, 2)$, where 1 and 2 refer to the base quartics $(\beta t)^4$ and $(\gamma t)^4$ of the fundamental involution, seemed to represent four collinear points of the curve when and only when the three nodes were adjacent in such a manner as to form an oscnode. In arriving at this conclusion it happened that, for each type of the curve examined, I chose as base quartics of the fundamental involution at least one catalectic quartic.‡ This accidental choice lead to the statement mentioned above. The failure of a later attempt to make the regular oscnodal invariants vanish under the condition that $\overline{12}^2(1, 2)$ should represent a line section of the curve caused the present investigation. In this paper, I shall discuss the form $\overline{12}^2(1, 2)$ and associated covariants and I shall develop several theorems which give new necessary and sufficient conditions for the oscnodal singularity.

2. *Certain Conditions.* I shall refer the curve to the tangents at the two points $t=0$ and $t=\infty$ and the line joining those points. This reference scheme assumes evidently that $t=0$ and $t=\infty$ are merely any two points of the curve such that the line of the curve at either does not pass through the other point. So this general reference triangle gives the curve as follows:

$$(1)\qquad \begin{cases} x_0 = a_0t^4 + b_0t^3 + c_0t^2, \\ x_1 = \qquad b_1t^3 + c_1t^2 + d_1t, \\ x_2 = \qquad\qquad c_2t^2 + d_2t + e_2, \end{cases}$$

where neither a_0 nor e_2 can be zero, and in order to avoid some

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† Neelley, *Effects of the oscnode upon covariant forms of the rational plane quartic curve*, this Bulletin, vol. 35, pp. 571–575.

‡ Neelley, *Compound singularities of the rational plane quartic curve*, American Journal of Mathematics, vol. 49 (1927), pp. 389–400.