ON THE DIRECT PRODUCT OF A DIVISION AND A TOTAL MATRIC ALGEBRA*

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This paper establishes certain theorems concerning an algebra A which is expressible as the direct product† of a division algebra D and a total matric algebra M. It is moreover not assumed that D and M are subalgebras of A. We let δ and n^2 represent the orders of D and M respectively. It follows that δn^2 is the order of A. We represent the modulus of A by be where b and e are the respective moduli of D and M. In agreement with the usual notation, we write

$$e = \sum e_{ii}, \ (i = 1, \cdots, n),$$

where e_{ij}, $(i,j = 1, \cdots, n)$, are the basal units of M.

For the proof of Theorem 1, we express the zero elements of algebras A, D and M by Z, z_d and z_m respectively. Thereafter we employ the symbol 0 without ambiguity. Since the elements of D and M are commutative with each other and a zero element of an algebra is unique, we have‡ $Z = z_d z_m$.

THEOREM 1. *If $dm = Z$, where d and m are any elements of D and M, respectively, then either $d = z_d$ or $m = z_m$.*

For, if $d \neq z_d$, it possesses an inverse d^{-1}. It follows that

$$bm = d^{-1}Z = d^{-1}z_d z_m = z_d z_m = Z.$$

Writing

$$m = \sum_{i,j=1}^{n} \alpha_{ij} e_{ij},$$

we have

$$\sum_{i,j=1}^{n} \alpha_{ij} b e_{ij} = Z.$$

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† Dickson, *Algebras and their Arithmetics*, p. 72.

‡ In the proof, let $Z = z_1 z_2$, where z_1 is in D and z_2 in M. Then

$$Z = Z \cdot z_d z_m = z_1 z_2 \cdot z_d z_m = z_1 z_d \cdot z_2 z_m = z_d z_m.$$