

Leçons sur les Nombres Transfinis. By W. Sierpinski. Paris, Gauthier-Villars, 1928. 240 pp.

The remarkable grace and ease with which this admittedly baffling subject is presented is sure to win for this book a warm welcome in many fields of mathematics. The book appears under the famous Borel collection of monographs, and habitual readers may be well assured that it is quite up to the standard set by previous contributors to this series. No previous knowledge of the subject treated or of the theory of sets is presupposed, and any mathematician who cares to familiarize himself with this field, which has assumed such large proportions in recent years because of its applications in the theory of functions and in the theory of sets, will find here this necessary material in a surprisingly accessible and readable form.

As is to be expected, much is found concerning the modern controversy between the so-called idealists and realists in mathematics, and the existence of that controversy pervades almost every page. The author himself, although lauded in the preface by Borel (who is a realist) as an idealist, which he undoubtedly is, nevertheless presents some of the arguments of the realists and takes pains to establish as much of the treatment as possible on a basis which would be acceptable even to the realists. For example, much stress—and in the reviewer's opinion, far too much stress—is placed on the fact that certain sets are *effectively* countable. The property of a given set M of being effectively countable, that is, of some person's knowing a law by means of which the elements of M may be enumerated, seems to the reviewer to be far too dependent on the human beings living at a given moment to be in any sense a mathematical property of a set. A given set M may be effectively countable tomorrow which is not so today; or worse yet, M may be effectively countable today and cease to be so tomorrow, for conceivably some person might discover a law by means of which the elements of M may be enumerated, and unfortunately die the next day leaving no record of his discovery. I do not like to think of such an elusive property as a mathematical property at all. A mathematical property should be something which is, once for all, either possessed or not possessed by a given set M , and whether or not M possesses this property certainly is independent of the particular group of mathematicians which happen to be alive at a given time. It should be well understood, however, that the reviewer's criticism is directed more at the realist than at the author of the book under review.

As indicated in the title and preface, this treatment is devoted entirely to a study of transfinite numbers in themselves, to the complete exclusion of their numerous applications in other branches of mathematics. It is divided quite naturally into two parts of approximately equal length, the first being devoted to cardinal numbers and the second to ordinal ones. In each part the first five chapters are concerned largely with propositions which can be established without the aid of the axiom of choice or Zermelo Postulate, and in the sixth and last chapter there are given theorems which can be proved only with the aid of this postulate, together with a discussion of some of the alternatives to the postulate and of some of the outstanding unsolved problems in the theory. In each case the arithmetic of and laws of operation with the respective classes